

A. K. GUPTA CLASSES

(JEE ADVANCE –SAMPLE PAPER)

Duration: 1Hour

Total Marks: 75

MATHEMATICS

Section-1:(only one option correct type)

1 .Let $S \equiv \{(x, y) : ||x| - 2| - 1| + ||y| - 2| - 1| = 1\}$. If S is made out of wire, then the lenth of wire required is

- a. $16\sqrt{2}$ b. $32\sqrt{2}$ c. $64\sqrt{2}$ d. $8\sqrt{2}$

2 .If fractional part of $\frac{1}{x}$ and x^2 for some $x \in (\sqrt{2}, \sqrt{3})$ are equal then value of $x^4 - \frac{3}{x}$ is

- a.3 b.5 c.6 d.7

3. If the equation $ax^2+bx+c=x^3+\frac{1}{x}$, ($a, b, c \leq 0$), has only real roots, then

- a . there can be positive roots
b. there can be maximum two negative roots.
c. when all the roots are integers, the value of $a + b + c$ is equal to -14.
d. none of these

4. If $2\sin x \sin y + 3\cos x \cos y = 7$, then the value of $\tan^2 x + 2\tan^2 y$ is equal to

- (a) 7 (b) 9 (c) 2 (d) none of these

5. Let the lines $(y-2)=m_1(x-5)$ and $(y+4)=m_2(x-3)$ intersect at right angle at (m_1, m_2) are parameter) P .If locus of P is $x^2+y^2+gx+fy+7=0$, then $f-g$ will equal to

- a.10 b.21 c. 24 d .none of these

6. If $x^5-x^3+x=a$, then minimum value of x^6 is

- a. a b. $2a-1$ c. $a+2$ d. $a-3$

7. Consider triangle ABC. The equation of BC is $y=x$. The centriod of triangle is $(8,3)$ and circum-centre is $(9,3)$. If the circum-radius of triangle is R then the value of $R/\sqrt{5}$ is

- a. 7 b. 3 c. 5 d. 8

8. $f(x) = \sin x^{\sin x}$, $\{x \in (0, \frac{\pi}{2})\}$ then range of $f(x)$ will be

- (a) $(0, 1)$ (b) $[(\frac{1}{e})^{\frac{1}{e}}, 1)$ (c) $(\frac{1}{e}, 1)$ (d) none of these

Section-2: (one or more options correct type)

9. If graph of $x y = 1$ is reflected in $y = 2x$ to give the $12x^2 + rxy + sy^2 + t = 0$, then

- A. $r=1, s=12, t=25$ B. $r=-1, s=12, t=1$
 C. $r=-7, s=-12, t=25$ D. $r+s=-19$

10. For the function $f(x) = (x^2 + bx + c)e^x$ and $g(x) = (x^2 + bx + c)e^x + e^x(2x + b)$. Which of the following holds good?

- a. if $f(x) > 0$, for all real $x \Rightarrow g(x) > 0$ b. if $f(x) > 0$, for all real $x \Rightarrow g(x) < 0$
 c. if $g(x) > 0$, for all real $x \Rightarrow f(x) > 0$ d. if $g(x) > 0$, for all real $x \Rightarrow f(x) < 0$

11. If $f(x) = \begin{cases} -x + 1, & x \leq 1 \\ -(x - 1)^2, & x \geq 1 \end{cases}$ then the solution $f(x) - f^{-1}(x) = 0$ is/are

- a. -1 b. 1 c. $\frac{1}{2}$ d. -2

12. A circle of radius r touches the curve $y^2 + 4x = 0$ at origin. The centre of the circle lies to the left of the origin and this circle lies completely within the curve. Then exhaustive range of r belongs to

- a. $(1, \frac{5}{2})$ b. $(0, 2)$ c. $(0, \frac{5}{2})$ d. $(0, 3)$

Section-3: Match the followings

13. If $f(x) = x^3 + ax^2 + bx + c = 0$ has three distinct inter roots and

$(x^2 + 2x + 2)^3 + a(x^2 + 2x + 2)^2 + b(x^2 + 2x + 2) + c = 0$ has no roots then

Column I

Column II

(a) Minimum value of a

(p) 0

(b) Minimum value of b

(q) 2

(c) Minimum value of c

(r) 3

d) Using , Minimum value of a, b and c . If the roots of

(s) -1

$f'(x) = k$, are equal, then $k =$

Section-4: paragraph based question (Question no.13-14)

Consider a rational function

$f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$ and a quadratic function $g(x) = (1+m)x^2 - 2(1+3m)x - 2(1+m)$, where m is a parameter.

14. If $\cot \cot^{-1} f(x) = k$, has exactly two distinct real solution then the integral value of k can be

- a. 0 b. -1 c. 1 d. 5.

15. If the range of the function $f(x)$ lies between the roots of $g(x) = 0$, then the number of integral values of m equals to

- a. 8 b. 6 c. 7 d. 9

Section-5: (Integer value correct type)

This section contains 5 questions. The answer to each question is single digit integer, ranging from 0 to 9

16. Let $2x + 3y = 0$ and $3x - 2y = 0$ intersect at A. Let another line whose equation is $ax + by + c = 0$ intersects the given two lines at B and C respectively. A circle touches AB at A and BC at D. It intersects AC in E. If $CE = 3, CD = 6$, then $BD = \dots$

17. The maximum value of $(x_1 + \sqrt{4 - x_2^2})^2 + (\sqrt{4 - x_1^2} - x_2)^2 - 11 = \dots$

18. If $x, y \in \mathbb{R}$, satisfy simultaneously the equation $\log x + \frac{\log(xy^8)}{\log^2 x + \log^2 y} = 2$, and

$\log y + \frac{\log(\frac{x^8}{y})}{\log^2 x + \log^2 y} = 0$, then value of $\frac{xy}{5}$ will be...

19. Let A denotes the value of expression $x^4 + 4x^3 + 2x^2 - 4x + 7$, where $x = \cot \frac{11\pi}{8}$ and B denotes the value of the expression

$\frac{1 - \cos 8\theta}{\tan^2 4\theta} + \frac{1 + \cos 8\theta}{\cot^2 4\theta}$, $\theta = 9^\circ$, then value of $\frac{AB}{2} = ?$

20. The maximum integral value of ' $a = 2n + 1$ ' for which the equation $2x^3 - 3x^2 - 12x + a = 0$

have three real and distinct roots then value of n will be equal to